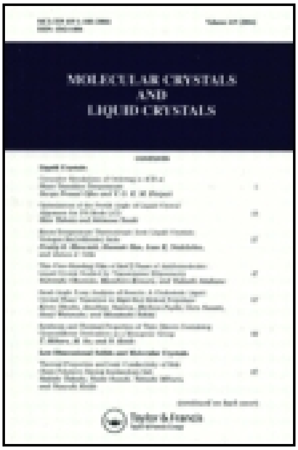

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On the Electrohydrodynamic Instability Limits of Nematics Under the Action of Sinusoidal Electric Fields

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In a recent paper[1] a method has been presented for the numerical computation of the electrohydrodynamic (EHD) instability limits and the corresponding director spatial wavenumbers (DSW) of a homogeneously aligned nematic cell, under the action of square-pulse electric field. The results obtained, using the physical constants of the room temperature nematic MBBA,[2–5] was in satisfactorily good agreement with experiments. The aim of this note is to present the results of the application of the model used in Ref. 1 to the case where the exciting electric field is a sinusoidal one. Such an application has already been published for the one-dimensional model,[6–8] as well as, for the two-dimensional one.[9] It is well known, however, that the calculations based on the one-dimensional model do not permit the explicit calculation of the DSW at the conduction mode (CM) thresholds.[10] On the other hand, the calculations of Ref. 9 are restricted to the CM of EHD instability under the, only approximately true, assumption that the director curvature is time independent. The method used here is free from these restrictions. It consists in numerically solving the coupled system of differential equations[1,6]

$$\dot{q} + a_1 q + b_1 E\psi = 0 \tag{1a}$$

$$\dot{\psi} + a_2\psi + b_2 Eq = 0 \tag{1b}$$

where q and ψ are the excess charge density and the director curvature, respectively, E is the instantaneous value of the applied electric field, and

a_1, a_2, b_1, b_2 are functions of the DSW, of E, of the thickness of the cell, and of the physical constants of the nematic material. They are defined in Ref. 1.

The system (1) is solved by the Runge–Kutta method. It turns out that,[7,8] in the CM of EHD instabilities, the solution, at the instability limits, is periodic in time, with q and ψ possessing only odd and even harmonics, respectively. The reverse holds for the dielectric mode (DM). Thus, for the CM case, the solution of the system (1) must, simultaneously, obey the conditions;

$$q(0) = -q(T/2) \tag{2a}$$

$$\psi(0) = \psi(T/2) \tag{2b}$$

where T is the period of the applied field. For the DM case the requirement is:

$$q(0) = q(T/2) \tag{3a}$$

$$\psi(0) = -\psi(T/2) \tag{3b}$$

For a particular value of T, one finds a continuous set of (E_{eff}, k) pairs for which the solutions of (1) meet the requirements (2), (or (3)), E_{eff} and k being the r.m.s. value of the applied field and the value of DSW, respectively. Among these pairs, those possessing the extremum values of E_{eff} correspond to the instability limits of the CM, (or DM). The associated values of k, are the DSW at these limits.

The results obtained by the outlined procedure are dependent on the

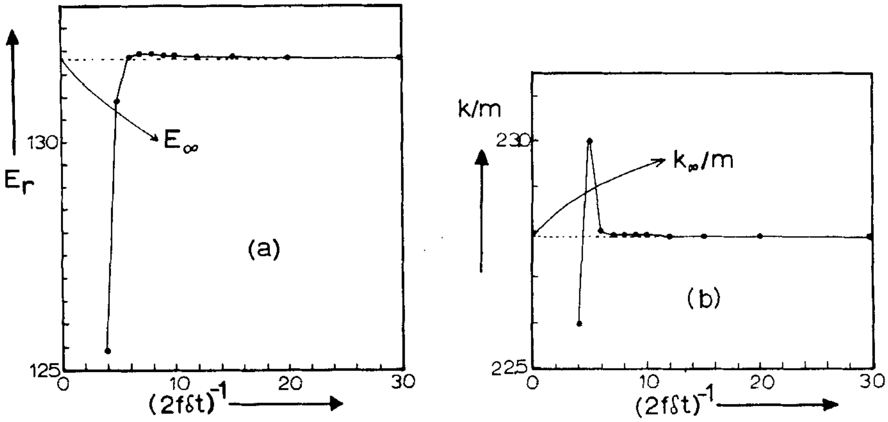

FIGURE 1 Typical step-of-integration dependence of computer results for (a) the field limit and (b) the DSW. The dependence is expressed in terms of the integer $N \equiv \pi/(\omega\delta t) = (2f\delta t)^{-1}$ which, actually, has been used in the calculations. Here $\omega = 2\pi f$, f being the frequency of the applied field. The lines connecting the points of the graphs have not the meaning of any functional dependence between the variables of the two axes, being drawn for convenience. Both plots have been obtained for DM using $f_r = 500$ cgs, $\sigma_r = 500$ cgs, and $H_r = 0$, (see text).

step-of-integration δt used in applying the numerical method. This dependence is illustrated in Figs. 1a and b, for a typical case. We considered a particular result, say for the DSW, as acceptable when

$$|k - k_\infty|/k_\infty < 10^{-3}$$

The meaning of E_∞ and k_∞ is obvious from the Figures 1a and b.

Figures 2, 3 and 4 illustrate the results of computer calculations. These are expressed in terms of the variables[1]

$$E_r \equiv E_{\text{eff}}/(K_{33}m^2)^{1/2}$$
$$f_r \equiv f/(K_{33}m^2)$$
$$\sigma_r \equiv \sigma_p/(K_{33}m^2)$$
$$H_r \equiv H/(K_{33}m^2)^{1/2}$$

and

$$k/m$$

where K_{33}: The bend curvature–elasticity constant.

$m \equiv \pi/L$, L: The thickness of the cell.

f: The frequency of the applied field.

σ_p: The parallel to the director conductivity of the nematic material.

H: The magnetic field applied parallel to the direction of the undisturbed director.

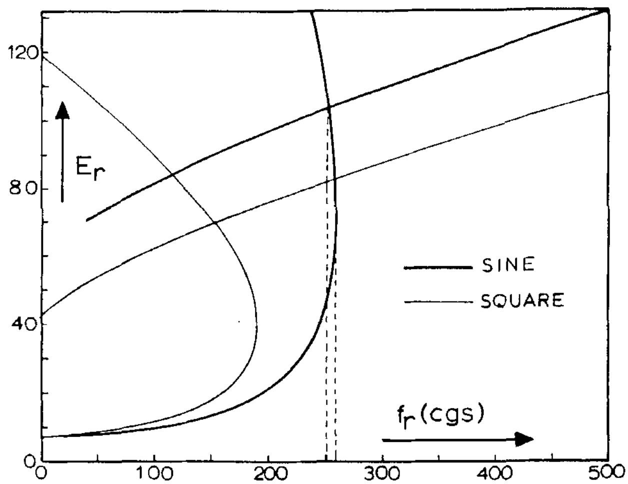

FIGURE 2 Computed plot of the frequency dependence of the limiting effective field under sinusoidal and square pulse electric fields. The CM and DM of both types of excitation are easily recognized. The two dashed vertical lines define the transition region of the sinusoidally excited system.

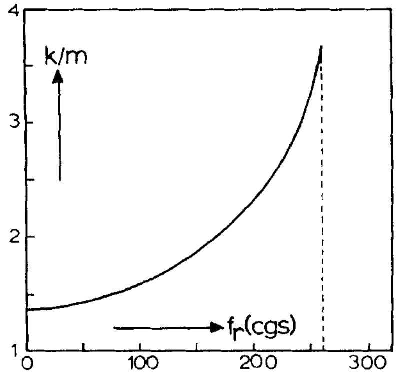

FIGURE 3 Computed plot of the frequency dependence of the threshold DSW, in units of *m*, at the CM. The dashed vertical line defines the cut-off frequency.

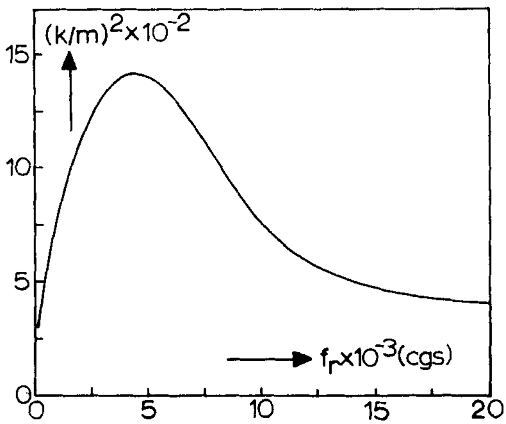

FIGURE 4 Computed plot of the frequency dependence of the DM threshold DSW, expressed in units of *m*. The dependence, in order to be easily comparable to Ref. 5, is given in terms of the square of the DSW.

The results, expressed in this way, depend on σ_r and H_r. The thickness dependence enters implicitly via these two parameters. The plots of Figures 2, 3 and 4 are obtained using the values:

$$\sigma_r = 500 \text{ cgs} \quad \text{and} \quad H_r = 0$$

Figure 2 gives the computed frequency dependence of the EHD instability limits together with the respective curve for the square pulse case. One easily recognizes the CM and DM parts of the curves.[11] The CM of EHD instability has two limits and appears up to a cut-off frequency. The DM has a single limit and does not exhibit any frequency cut-off. Part of the upper limit curve of the CM constitutes boundary between the stability and instability regions of the $E_r - f_r$ diagram. That is, there exists a transition region[7] which, however, is much narrower than that of the square-pulse case. (One can calculate points of the upper limit curve of the CM down to zero frequency, but this would require much computer time,[12] without any gain of information which could be tested experimentally, because of the dynamic scattering persisting at this region of the $E_r - f_r$ diagram.[13] The same holds for the low frequency part of the DM curve).

It is known that,[6,7] the CM is characterized by a voltage threshold whereas the DM by a field threshold. This fact, which is not examined here and which is approximately true, will be discussed in a future work on the dependence of the EHD limits and the DSWs on the thickness of the cell, the conductivity of the nematic material and the strength of the stabilizing, magnetic or high frequency electric, field.

The results given by Figure 2 agree, in their general features, with the calculations based on the one-dimensional model,[7,8] as well as, with the experimental curve of Ref. 11. The only point of disagreement is that no transition region is found in this reference for the sinusoidal excitation case. That a transition region may exist under sinusoidal excitation of negative dielectric anisotropy nematics is already known.[14] Therefore further experimental and numerical-analytical work is required to clarify this point.

The curves of Figures 3 and 4 give the computed frequency dependence of the threshold DSW (in units of m), for the CM and DM, respectively. The CM DSW, and also its frequency gradient, increase monotonically with frequency up to its cut-off value. This general behavior agrees with the experiments of Refs. 15 and 16. The DM DSW increases with frequency until it reaches a maximum, which is of the order of the Debye length of the material,[17] thereby decreasing and tending to some nonzero limit. This behavior results because of the diffusion currents developed inside the nematic.[17] From a comparison of the curve of Figure 4 to the respective experimental curve of Ref. 5, where this behavior is compared to the predictions of the one dimensional model,[17] it follows that our method gives results much closer to the experiments.

To establish the rigorous validity of our method of calculating instability limits and DSWs, a series of measurements are being done in our laboratory. The results will be published as soon as possible.

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